3.5: Derivative of Trigonometric Functions

Let $f(x)=\sin x$. Then

$$f'(x)=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}=\lim_{h\to 0}\frac{\sin x\cosh+\cos x\sin h-\sin x}{h}$$

$$=\lim_{h\to 0}\frac{\sin x(\cos h-1)+\cos x\sin h}{h}=\lim_{h\to 0}\frac{0+\cos x\sin(h)}{h}=\cos x.$$

Thus (6) $\frac{d}{dx}(\sin x)=\cos x$

Let $f(x)=\cos x$. Then

$$f'(x)=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}=\lim_{h\to 0}\frac{\cos x\cos h-\sin x\sin h-\cos x}{h}$$

$$=\lim_{h\to 0}\frac{\cos x(\cos h-1)}{h}-\lim_{h\to 0}\sin x\cdot\frac{\sin h}{h}=-\sin x.$$

Thus ~~#~~ (7) $\frac{d}{dx}(\cos x)=-\sin x$.

Use the quotient rule to find ~~th the~~ $\frac{d}{dx}(\tan x)$.

$$\frac{d}{dx}(\tan x)=\sec^2 x.$$

Ex(1): Find the derivative of $y=5e^x+\cos x$ and $f(x)=\sin x\cos x$.

$$y'=5e^x-\sin x,\quad f'(x)=\cos^2 x-\sin^2 x.$$

Find the derivatives of $\csc x$, $\sec x$, $\cot x$.

$$\frac{d}{dx}(\csc x)=-\csc x\cot x,\quad \frac{d}{dx}(\sec x)=\sec x\tan x,\quad \frac{d}{dx}(\cot x)=-\csc^2 x.$$

Ex(2): Find y'' when $y=\sec x$.

$$y''=\sec^3 x+\sec x\tan^2 x.$$

Derivative of $\ln x$: Let $f(x) = \ln x$

$$\frac{d}{dx} f'(x) = \lim_{h \to 0} \frac{\ln(x+h) - \ln x}{h} = \ \overset{?}{\vdots} \ = \frac{1}{x}.$$

We will see how to show this later on.

(3) $\frac{d}{dx}(\ln x) = \frac{1}{x}$. $\frac{d}{dx}(\log_a(x)) = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right) = \frac{1}{\ln a \cdot x}$.